

<p>Grade Level/Course:</p> <p>Algebra 1</p>
<p>Lesson/Unit Plan Name:</p> <p>Using The Functional Notation Form of Point-Slope to Write Linear Equations and Solve Application Problems</p>
<p>Rationale/Lesson Abstract:</p> <p>Given data oftentimes does not include initial conditions under which one can use slope-intercept form. Using point-slope in functional notation form, $f(x) = m(x - x_1) + y_1$, enables the user to write and simplify an equation to find the initial condition as well as determine future results. In addition, this form invites comparisons to other equations like the vertex form of a parabola.</p> <p>This lesson is taught AFTER lessons on identifying rate of change in various ways and use of slope-intercept form of a function.</p> <p>Objective: Students will write equations in point-slope form $[f(x) = m(x - x_1) + y_1]$ and use to solve application problems. Given conditions include: 1). One data point and rate of change, 2). Two data points, 3). A graphed line, 4). A table of data.</p>
<p>Timeframe:</p> <p>2 days</p>
<p>Common Core Standard(s):</p> <p>High School Algebra F.LE.2--Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).</p>

Instructional Resources/Materials:

- Copies of handouts for both days (p. 8, 9, 11, 12)
- Copies of exit tickets for both days (p. 10 and 13)

Activity/Lesson:**DAY 1****Do Now:** Simplify these expressions: (5 min)

$$(x - 20) + 120$$

$$= x - 20 + 120$$

$$= x + 100$$

$$-7(x - 5) + 89$$

$$= -7x + 35 + 89$$

$$= -7x + 124$$

$$\frac{5}{2}(x - 8) + 55$$

$$= \frac{5}{2}x - \frac{5}{2}(8) + 55$$

$$= \frac{5}{2}x - 20 + 55$$

$$= \frac{5}{2}x + 35$$

Notes:

Teacher leads discussion as students watch or take notes: (3-7 minutes)

Start with slope formula: $\frac{y_2 - y_1}{x_2 - x_1} = m$ Solve for y_2 .

$$\frac{y_2 - y_1}{x_2 - x_1} = m$$

$$(x_2 - x_1) \frac{y_2 - y_1}{x_2 - x_1} = m(x_2 - x_1)$$

-----Multiply both sides by denominator to clear the fraction.

$$(x_2 - x_1) \frac{y_2 - y_1}{x_2 - x_1} = m(x_2 - x_1)$$

$$\text{-----Giant 1!!} \quad \frac{(x_2 - x_1)}{(x_2 - x_1)} = 1$$

$$y_2 - y_1 = m(x_2 - x_1)$$

$$y_2 - y_1 + y_1 = m(x_2 - x_1) + y_1$$

-----Add y_1 to both sides.

$$y_2 = m(x_2 - x_1) + y_1$$

$$f(x) = m(x - x_1) + y_1$$

-----Rewrite y_2 as the output of the function, $f(x)$, and x_2 as the input value, x .

This is the equation of a linear function in point-slope form.

$$f(x) = m(x - x_1) + y_1$$

slope (rate of change)

data point (x_1, y_1)

“Let’s use this equation to describe real-world applications mathematically (on handout):”
Teacher directs students to handout.

We Do: Ashley deposits \$30 into her bank account each month. After 12 months she has \$550. Write an equation in point-slope form representing Ashley’s situation. (10 min)

What do we know? Rate of change: \$30 per month. Data point: (Month 12, \$550)

Write the point-slope form: $f(x) = m(x - x_1) + y_1$

Substitute values:

$$f(x) = 30(x - 12) + 550$$

How much did she have in the bank initially?

Simplify the equation: $f(x) = 30x - 360 + 550$

$$f(x) = 30x + 190$$

Notice that the equation is in slope-intercept form [$f(x) = mx + b$], which tells you the initial condition, in this case, how much money Ashley initially had in the bank before she started depositing \$30 per month. (Students can also let $x = 0$.)

So, Ashley started with \$190 in her bank account.

If she continues to deposit money at the same rate, how much will she have after 20 months?

With the simplified function, substitute the input value of 20 for x:

$$f(x) = 30x + 190$$

$$f(20) = 30(20) + 190$$

$$f(20) = 600 + 190$$

$$f(20) = 790$$

After 20 months she will have \$790 in her account.

“Now it’s your team’s turn to try. Team member #2, please ask someone to read the next problem. Ready, go!” (20 min for teamwork)

Teamwork: Tiffany and Mansi set up a lemonade stand and sell cups of lemonade at a rate of 7 cups per hour. After 5 hours there are 89 cups left. Write an equation in point-slope form that would represent this situation. (*Hint: Are the number of cups increasing or decreasing? How will this be shown in your rate of change?*)

Given: Rate of change: -7 cups per hour Data point: (5 hours, 89 cups)

$$f(x) = m(x - x_1) + y_1$$

$$f(x) = -7(x - 5) + 89$$

Find how many cups the young entrepreneurs started with:

$$f(x) = -7(x - 5) + 89$$

$$f(x) = -7x + 35 + 89$$

$$f(x) = -7x + 124$$

They started with 124 cups.

If they continue to sell at the same rate, how many cups will they have after 15 hours?

$$f(x) = -7x + 124$$

$$f(15) = -7(15) + 124$$

$$f(15) = -105 + 124$$

$$f(15) = 19$$

They will have 19 cups left after 15 hours of selling lemonade.

The next team problem challenges students with a fractional rate of change. Students can refer to the Do Now, which has the expression simplified.

The challenge problem shows a graph. Students must first determine the rate of change. They should have practiced this skill already (slope formula or slope triangle).

[Solutions: $f(x) = \frac{1}{2}(x - 1) - 3$ or $f(x) = \frac{1}{2}(x + 1) - 4$]

Allow 5 minutes for independent practice problem.

[Solution: $f(x) = 50(x - 7) - 312$. She borrowed \$662 from the bank.]

Assessment: Allow 5 minutes for exit ticket.

After getting straight A's on her report card, Mary notices that her Facebook adds 3 new friends every week. After 12 weeks, she has 100 friends. Write an equation in point-slope form that would represent this situation.

$$f(x) = 3(x - 12) + 100$$

How many Facebook Friends did she have before her fabulous report card?

She had 64 Facebook Friends before her report card.

Activity/Lesson continued:

DAY 2

Do Now: (5 min)

Find the rate of change from these data points (Month 3, \$120) and (Month 10, \$400)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{400 - 120}{10 - 3}$$

$$m = \frac{280}{7}$$

$$m = \frac{40}{1} \quad \text{Read: \$40 per month increase.}$$

Teacher directs students to handout.

We Do: (10 min) *What is different about this problem compared to yesterday's problems?*

Ali decided to increase the number of plants in his backyard garden. After 8 weeks he has 72 plants. After 20 weeks he has 120 plants. Write an equation in point-slope form that would represent this situation.

Given: Data points: (8, 72) and (20, 120)

Find the rate of change:

OR

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{120 - 72}{20 - 8}$$

$$m = \frac{48}{12}$$

$$m = 4$$

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{72 - 120}{8 - 20}$$

$$m = \frac{-48}{-12}$$

$$m = 4$$

Write the point-slope equation and substitute either data point. Teacher could ask half the class to substitute one point and the other half to substitute the other point.

$$f(x) = m(x - x_1) + y_1$$

$$f(x) = 4(x - 20) + 120$$

OR

$$f(x) = 4(x - 8) + 72$$

How many plants did he start with?

Simplify the expression to slope-intercept form to find the initial condition:

$$f(x) = 4(x - 20) + 120$$

OR

$$f(x) = 4(x - 8) + 72$$

$$f(x) = 4x - 80 + 120$$

$$f(x) = 4x - 32 + 72$$

$$f(x) = 4x + 40$$

$$f(x) = 4x + 40$$

He started with 40 plants.

He started with 40 plants.

If he continues to increase the number of plants at the same rate, how many plants will he have after 30 weeks?

$$f(x) = 4x + 40$$

$$f(30) = 4(30) + 40$$

$$f(30) = 120 + 40$$

$$f(30) = 160$$

He will have 160 plants after 30 weeks.

"Now it's your team's turn to try. Team member #4, please ask someone to read the next problem. Ready go!" (20 min for teamwork)

Teamwork: After 2 hours, a climber is at an altitude of 400 feet. After 6 hours, she is at 700 feet. Write an equation in point-slope form that would represent this situation.

Given two data points: (2 hours, 400 feet) and (6 hours, 700 feet)

$$m = \frac{y_2 - y_1}{x_2 - x_1}$$

$$m = \frac{700 - 400}{6 - 2}$$

$$m = \frac{300}{4}$$

$$m = 75$$

OR

$$f(x) = m(x - x_1) + y_1$$

$$f(x) = 75(x - 2) + 400$$

$$f(x) = m(x - x_1) + y_1$$

$$f(x) = 75(x - 6) + 700$$

At what altitude did she start climbing?

$$f(x) = 75(x - 2) + 400$$

OR

$$f(x) = 75(x - 6) + 700$$

$$f(x) = 75x - 150 + 400$$

$$f(x) = 75x - 450 + 700$$

$$f(x) = 75x + 250$$

$$f(x) = 75x + 250$$

She started at an altitude of 250 feet.

She started at an altitude of 250 feet.

If she continues to climb at the same rate, how high will she be after 10 hours?

$$f(x) = 75x + 250$$

$$f(x) = 75x + 250$$

$$f(10) = 75(10) + 250$$

$$f(10) = 750 + 250$$

$$f(10) = 1000$$

She will have climbed to an altitude of 1000 feet after 10 hours.

The next team problem requires students to recognize that they can represent 3 for March and 10 for October. They can refer to the Do Now to see the rate of change worked out.

The last two teamwork problems have a graphed line and a table, where students must first determine the rate of change. Graph problem solutions: $f(x) = 4(x + 2) + 1$ or $f(x) = 4(x + 3) - 3$

One possible table problem solution: $f(x) = 7(x - 2) + 60$

Independent practice has a graphed line. Allow 5 minutes.

[Solutions: $f(x) = \frac{1}{2}(x - 1) - 3$ **or** $f(x) = \frac{1}{2}(x + 1) - 4$ **]**

Assessment: Exit Ticket – Allow 5 minutes

The nightly cost for 4 people to rent a house in Mendocino is \$220. For 6 people it costs \$280.

Write an equation in point-slope form that would represent this situation.

$$f(x) = 30(x - 4) + 220 \quad \text{OR} \quad f(x) = 30(x - 6) + 280$$

Find the cost for 10 people.

It would cost \$400 for 10 people.

Possible Next Day's Lesson:

Teachers can create word problems where there is not enough information available to create a linear equation. Students practice recognizing what information would be needed to develop a linear equation, then create a statement with the needed information and solve the problem.

Example: Mark had \$300 in his savings account in April. How much will he have in December? How much did he have in January?

Students could recognize that the rate of change is missing OR that an additional data point is missing. They can make up an additional statement:

"He deposits \$45 per month." OR "He had \$600 in June."

They would then write the equation and determine the answers to the questions.

MA# _____ : Writing Equations in Point-Slope Form

Name: _____

(Given a point and the slope)

If you have a point of data and the rate of change (slope) you can write a linear function in Point-Slope Form:

Let's take a look at an example together:

We Do: Ashley deposits \$30 into her bank account each month. After 12 months she has \$550. Write an equation in point-slope form representing Ashley's situation.

What do we know?

Write the point-slope form:

Substitute values:

How much did she have
in the bank initially?

If she continues to deposit money at the same rate, how much will she have after 20 months?

Teamwork: Tiffany and Mansi set up a lemonade stand and sell cups of lemonade at a rate of 7 cups per hour. After 5 hours there are 89 cups left. Write an equation in point-slope form that would represent this situation. (*Hint: Are the number of cups increasing or decreasing? How will this be shown in your rate of change?*)

Find how many cups the young entrepreneurs started with:

If they continue to sell at the same rate, how many cups will they have after 15 hours?

124 cups

19 cups

Mr. Gutierrez got a salary raise at Dell Computers. Since then, he decided to buy 5 pairs of shoes of shoes every 2 months. After 8 months he had 55 pairs of shoes. Write an equation in point-slope form that would represent this situation.

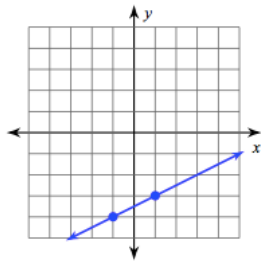
How many pairs of shoes did he have before his raise?

35 pairs

If he continues to purchase shoes at the same rate, how many pairs of shoes will he have after 12 months?

65 pairs

Challenge: Write an equation in point-slope form of the line shown:



Independent Practice: Daeja borrowed money from a bank to start a small friendship bracelet business. She pays the bank \$50 per month and after 7 months she still owes \$312. Write an equation in point-slope form that would represent this situation. (*Hint: Owning money is negative!*)

How much money did she borrow from the bank?

<p>EXIT TICKET Name: _____</p> <p>After getting straight A's on her report card, Mary notices that her Facebook adds 3 new friends every week. After 12 weeks, she has 100 friends. Write an equation in point-slope form that would represent this situation.</p> <p>How many Facebook Friends did she have before her fabulous report card?</p>	<p>EXIT TICKET Name: _____</p> <p>After getting straight A's on her report card, Mary notices that her Facebook adds 3 new friends every week. After 12 weeks, she has 100 friends. Write an equation in point-slope form that would represent this situation.</p> <p>How many Facebook Friends did she have before her fabulous report card?</p>
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MA# _____ : Writing Equations in Point-Slope Form

Name: _____

(Given two points of data)

Yesterday, we used the Point-Slope Form of a line to write equations.

We Do: *What is different about this problem compared to yesterday's problems?*

Ali decided to increase the number of plants in his backyard garden. After 8 weeks he has 72 plants. After 20 weeks he has 120 plants. Write an equation in point-slope form that would represent this situation.

How many plants did he start with?

If he continues to increase the number of plants at the same rate, how many plants will he have after 30 weeks?

Teamwork: After 2 hours, a climber is at an altitude of 400 feet. After 6 hours, she is at 700 feet. Write an equation in point-slope form that would represent this situation.

Simplify the equation:

At what altitude did she start climbing?

250 feet

If she continues to climb at the same rate, how high will she be after 10 hours?

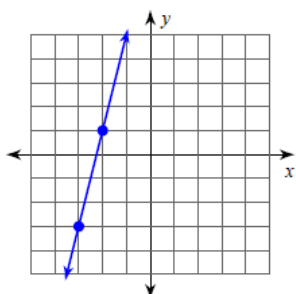
1000 feet

Zalma's stock investment in March was \$120. In October, it was \$400. Write an equation in point-slope form that would represent this situation.

If this investment increases at the same rate, how much money will she have in December?

\$480

Write an equation in point-slope form of the line shown:



Write an equation in point-slope form that represents the data in the table shown below:

hours	miles
2	60
5	81
8	103
11	124

Independent Practice: Write an equation in point-slope form of the line shown:

